FIG. 1 Wavelet Tiling of an N-Point Digital t-f Space

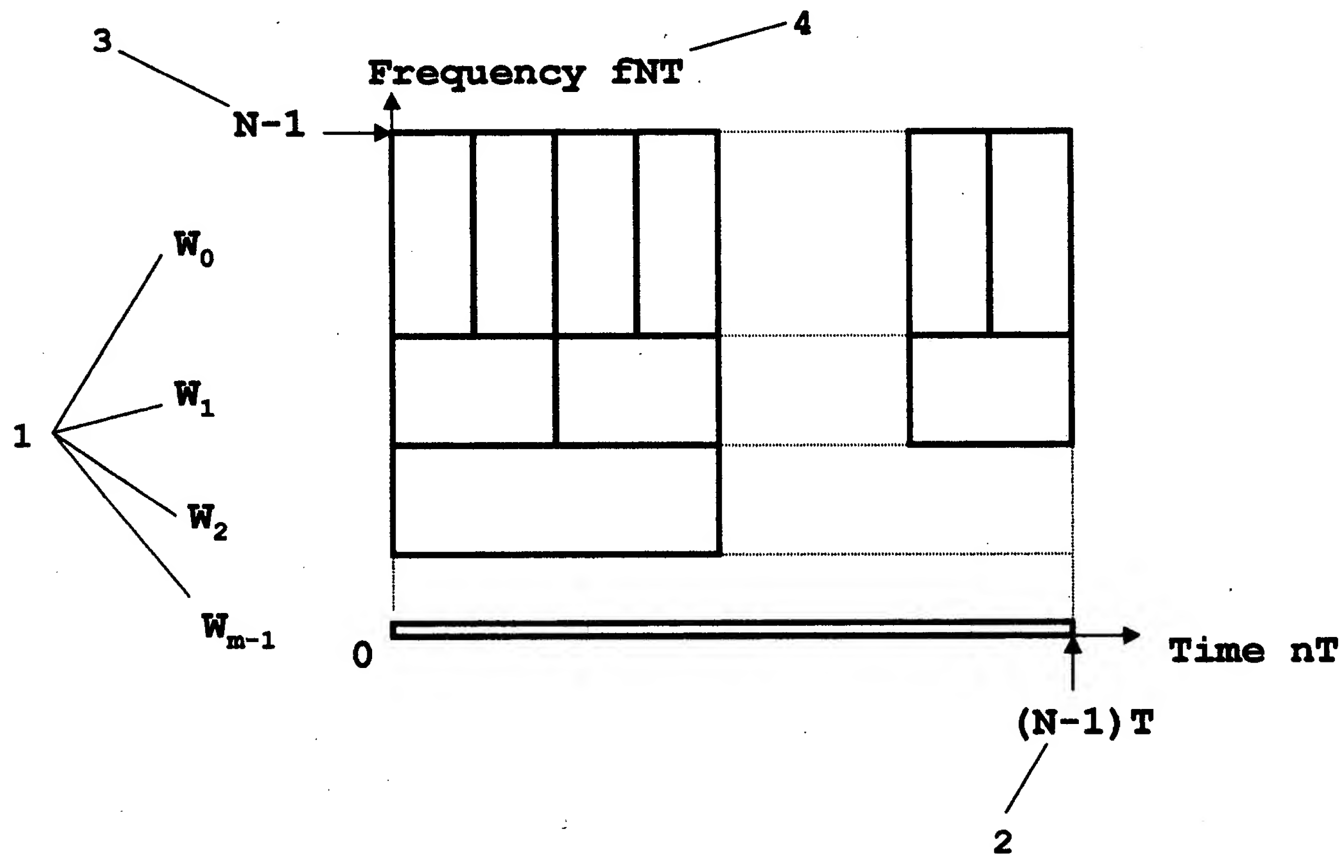

FIG. 2 Wavelet Iterated Filter Bank for Tiling t-f Space in FIG. 1

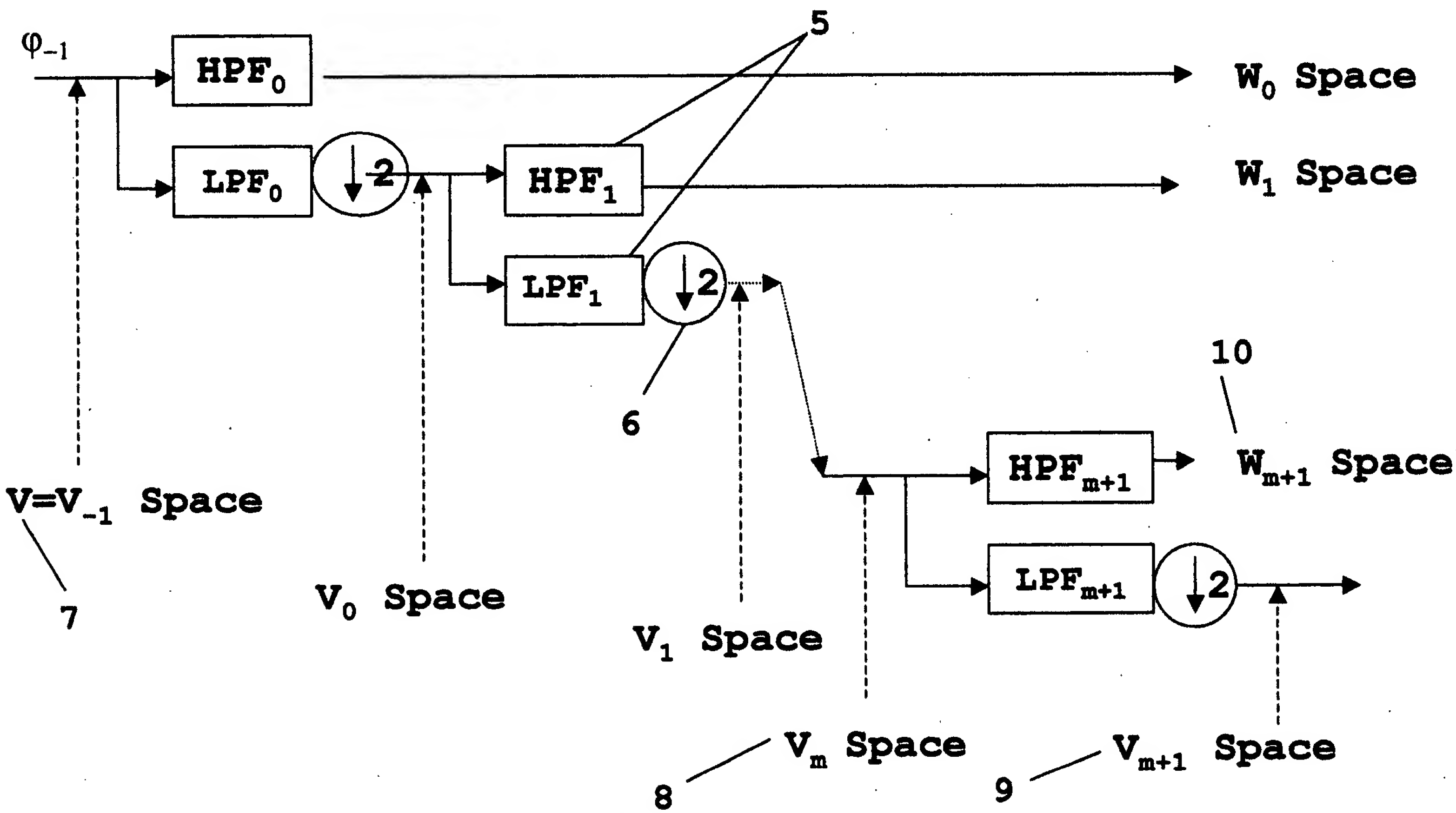

FIG. 3 PSD Requirements for Communications

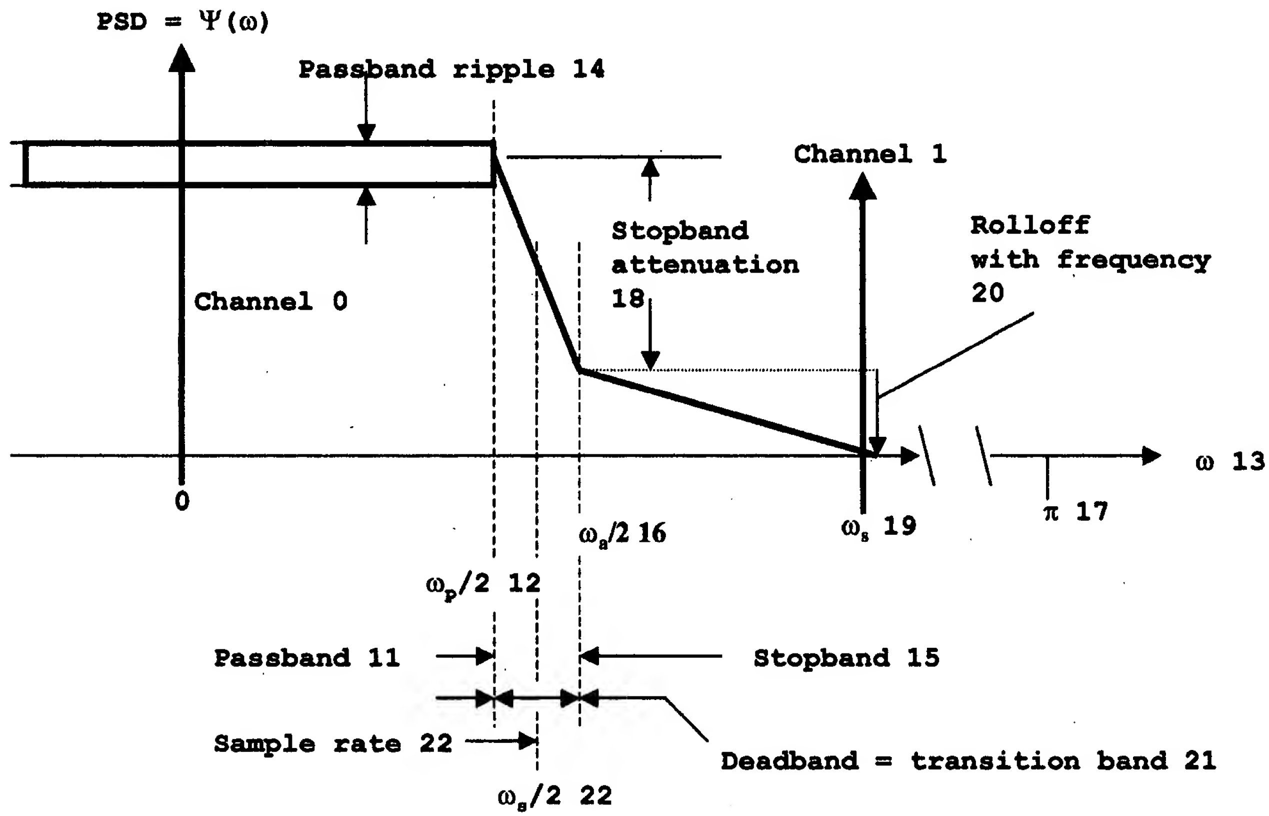

FIG. 4 LS Metrics and Cost Function

Design Parameters

Passband Eq. 23
Stopband Eq. 23
Deadband Eq. 24

ISI Eq. 25
ACI Eq. 26

LS Cost equation

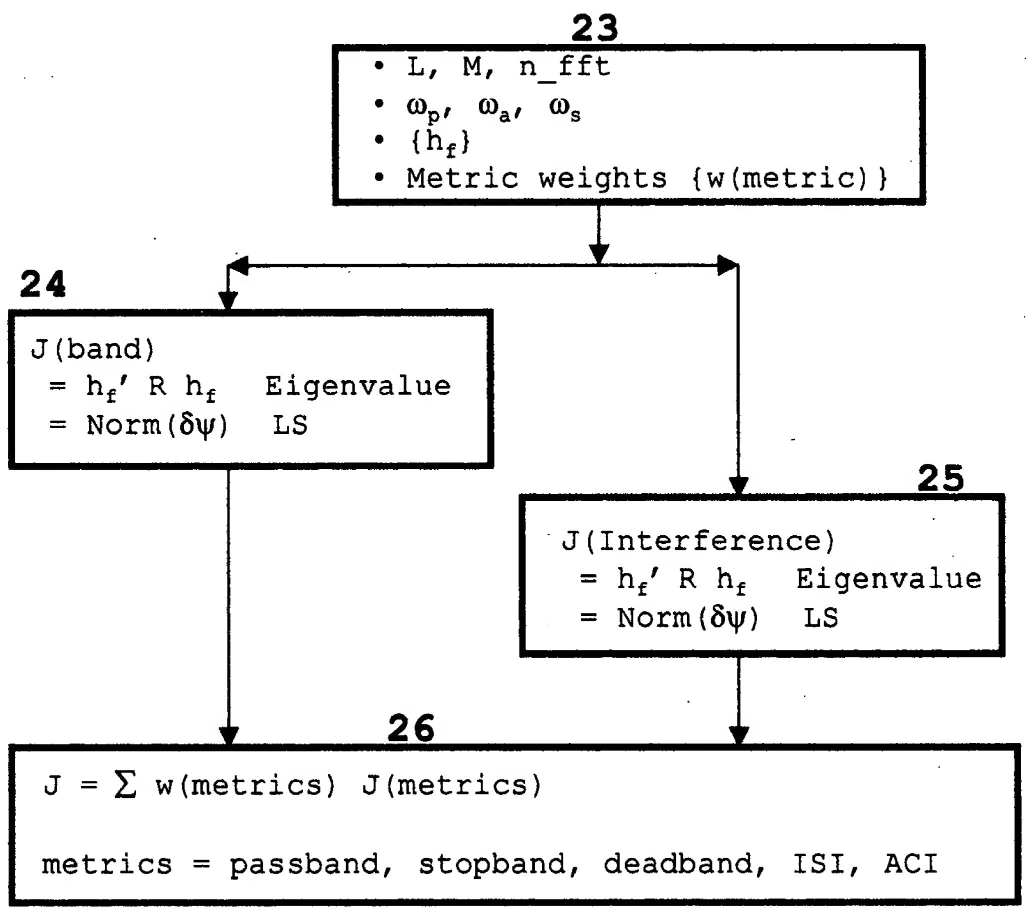

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FIG. 5 LS Recursive Solution Algorithm

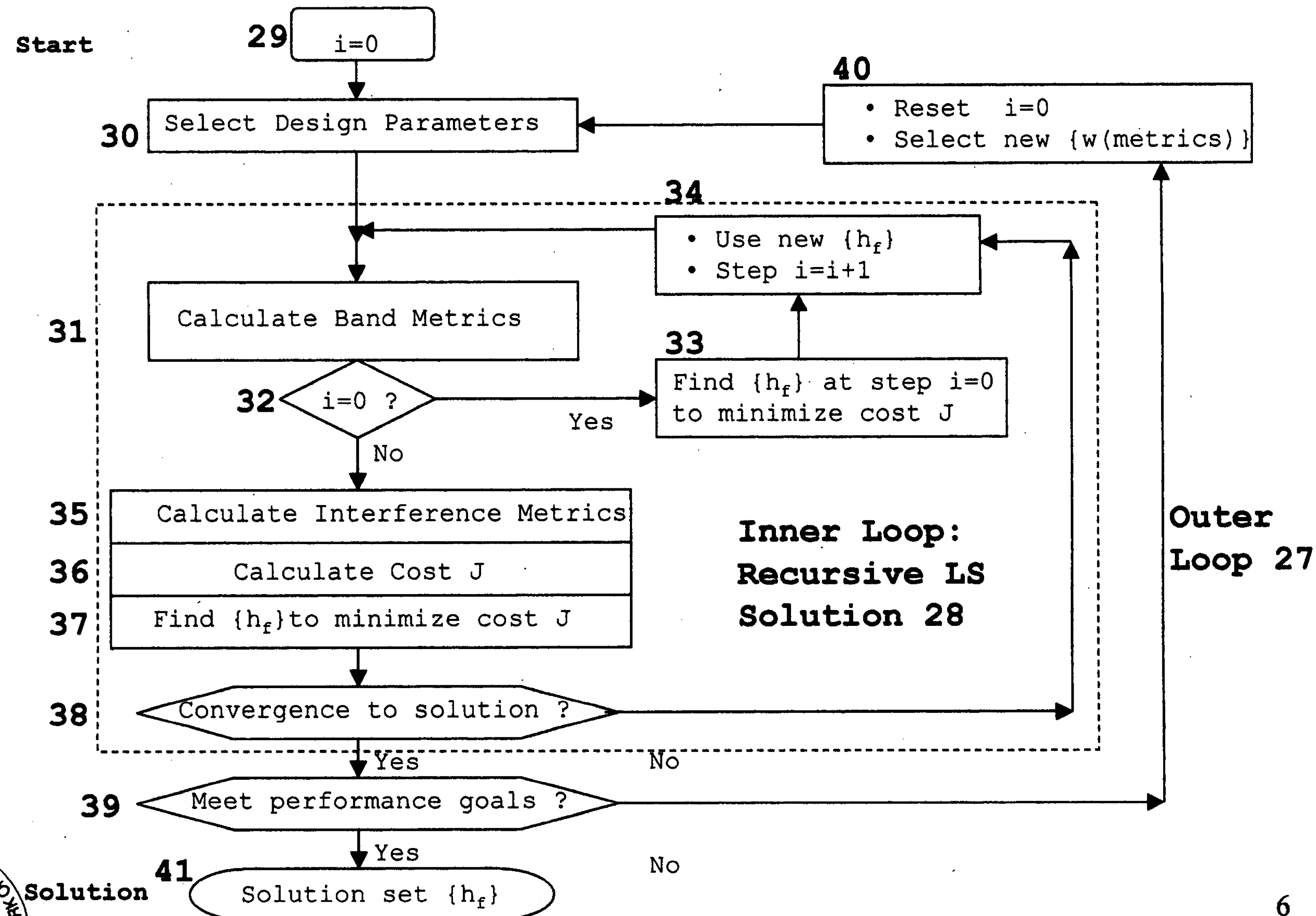

FIG. 5A LS RECURSIVE DESIGN ALGORITHM IN MATLAB 5.0 CODE TO DESIGN:

- **MOTHER WAVELET IN FIG. 6**
- **NEW WAVELET FROM MOTHER WAVELET**
- **PERFORMANCE DATA AND PLOTS**

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```
% STEP 1 DESIGN PARAMETERS
%==============================================================
% STEP 1.1 SCENARIO PARAMETERS
%====
M            = 16;       % Wavelet sample interval
L            = 16;       % nominal Wavelet length in units of M
N            = M*L+1 ;   % Wavelet length N =ML+1
fs           = 1;        % normalized channel spacing
                         % = normalized Wavelet sample rate
fp           = 0.8864;   % normalized channel passband
n_f          = 16        % number of design harmonics
n_fft        = 1024 ;    % FFT size for spectrum centered at 0
ebno         = 6.0 ;     % dB, Eb/No
x_imbal_aci  = 6.0 ;     % dB, channel-to-channel imbalance
%==============================================================
% STEP 1.2 DERIVED PARAMETERS
twopi        = pi*2 ;    % definition
nc           = M;        % maximum number of channels allowed
nfft_wsr     = (n_fft/M) % 0.5 * Wavelet sample rate
f_pass       = fp/(M*fs)            % wp/2pi edge of passband
f_stop       = (2-fp)/(M*fs)        % ws/2pi edge of stopband
nfft_pass    = floor(f_pass*n_fft) % edge of passband
nfft_stop    = floor(f_stop*n_fft) % edge of stopband==========
% STEP 1.3 OPTIMIZATION PARAMETERS
%====
n_iteration = 10;        % number of iterations for LS design
alpha_1      = 1.e-2     % weighting for passband
alpha_2      = 0.80      % weighting for stopband
alpha_3      = 2.e-3     % weighting for ISI
alpha_4      = 0.5       % weighting for ACI
alpha_5      = 0         % weighting for deadband
%==============================================================
```

FIG. 5B

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```
% STEP 2 INITIALIZATION CALCULATIONS
%==============================================================
% STEP 2.1 WAVELET LENGTH PARAMETERS
%====
nodd= fix( N/2 );
nodd = N - 2 * nodd ;
if ( nodd == 1)
  m = (N - 1 ) /2   %  N is odd
  nrow = m+1;
else
   m = N/2 ;   %    N is even
   nrow = m;
end
%==============================================================
% STEP 2.2 MATRIX "bw_matrix" MAPS WAVELET FREQUENCY DESIGN
%         HARMONICS INTO WAVELET TIME RESPONSE
%====
bw_matrix = zeros(m,n_f);
for i_r=1:m            % freq
   ang = 2*pi* rem( (i_r)*(0:n_f-1)/(N-1),1); % time
   bw_matrix(i_r + 1, :) = 2 * cos(ang);
end
bw_matrix(1,:) = ones(1,n_f);
%==============================================================
% STEP 2.3 FUNCTION "pmn" CALCULATES PASSBAND, STOPBAND LS
%         ERROR MATRICES FOR THE METRICS J(PASS), J(STOP) IN
%         EQ. 23 AND FUNCTION "pmn_d" CACULATES ERROR MATRIX
%         FOR J(DEAD)IN EQ. 24
%==============================================================
% STEP 2.4 MATRIX "c_matrix" USED FOR ISI,ACI LS ERROR METRICS
%         J(ISI)IN EQ. 25 AND J(ACI) IN EQ. 26
%===
au=eye(m+1,m+1);     % identity matrix
bb=rot90(au);        % rotation
c_matrix=[bb; flipud(bb(1:m,1:(m+1)))]/2;
c_matrix(m+1,1)=c_matrix(m+1,1)*2;
%==============================================================
% STEP 2.5 PASSBAND,STOPBAND, WAVELET SAMPLE RATE TEMPLATES
%====
%====set up passband and stopband templet
v_1 = 1:nfft_pass+1;
v_2 = nfft_pass+2:nfft_stop;
v_3 = nfft_stop+1:nfft_stop+nfft_pass;
hw_ref= [zeros(size(v_1))  -110*ones(size(v_2)) ...
         zeros(size(v_3))];
%====set up wavelet sample rate templet
v_1b = 1:nfft_wsr;
v_2b = nfft_wsr+1:nfft_wsr+1;
v_3b = nfft_wsr+2:nfft_wsr+nfft_pass+nfft_stop;
hw_wsr= [-110*ones(size(v_1b))  zeros(size(v_2b)) ...
      -110*ones(size(v_3b))];
%==============================================================
```

Replacement Sheet

FIG. 5C

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% STEP 3 PASSBAND, STOPBAND, DEADBAND LS ERROR MATRICES

```
%============================================================
% STEP 3.1 J(PASSBAND) LS ERROR MATRIX "passband"
%====
omega_l = 0.0 * pi;
omega_u = f_pass  * pi ;   % 0.0554
an=ones(1,nrow);
passband = pmn( omega_l, omega_u, N, an) ;
%============================================================
% STEP 3.2 J(STOPBAND) LS ERROR MATRIX "stopband"
%====
omega_l = f_stop * pi;   %
omega_u = pi ;
an=zeros(1,nrow);
stopband = pmn( omega_l, omega_u, N, an) ;
%============================================================
% STEP 3.3 J(DEADBAND) LS ERROR MATRIX "deadband"
%====
omega_l = f_pass  * pi;   %
omega_u = f_stop * pi;
an=ones(1,nrow);
%deadband = pmn_d( omega_l, omega_u, N, an) ;
deadband = zeros(nrow,nrow);
%============================================================
% STEP 3.4 WEIGHTED LS ERROR MATRIX "p_total" FOR THE WEIGHTED
%          SUM OF J(PASSBAND), J(STOPBAND), J(DEADBAND)
%====
p_total= alpha_1*passband+alpha_2*stopband+alpha_5*deadband;
%============================================================
% STEP 3.5 CONVERT LS ERROR MATRIX IN TIME "p_total" TO LS
%          ERROR MATRIX IN FREQUENCY "pw_t"
%====
pw_total = bw_matrix'*(p_total*bw_matrix);
pw_t = pw_total;
%============================================================
```

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% STEP 4 ITERATIVE EIGENVALUE SOLUTION

```
%====
hn_data          = [];
hw_data          = [];
err_LS           = [];
loss_LS          = [];
for i_iteration = 1:n_iteration
    i_iteration
%====
```

Replacement Sheet

FIG. 5D

```
%===========================================================
% STEP 4.1 FOR EACH ITERATION "i_iteration" FIND EIGENVECTOR
%          IN FREQUENCY THAT MINIMIZES THE COST FUNCTION J IN
%          EQ. 27 WHOSE LS  ERROR MATRIOX IS  "pw_t"
%====
eig_val = eig(pw_t);
  [eig_vec eigval] = eig(pw_t);
  [eigval_min,min_index] = min(eig_val);
%===========================================================
% STEP 4.2 MAP EIGENVECTOR INTO:
%          - WAVELET FREQUENCY DESIGN HARMONICS "hw_eig"
%          - WAVELET IMPULSE RESPONE IN TIME "hn"
%====
b_vector = bw_matrix * eig_vec(:,min_index);
hw_eig   = eig_vec(:,min_index);
hw_eig(1)= 2*hw_eig(1);
hw_max   = max(hw_eig);
hw_eig   = hw_eig/hw_max;
if ( nodd == 1)    % N is odd
   hn(1:m) = 0.5*b_vector((m+1):-1:2);
   hn(m+1) = b_vector(1);
   hn(m+2:N) = hn(m:-1:1);
else
   hn(m:-1:1)= 0.5 * b_vector(1:m);
   hn( m+1:1:2*m) = hn(m:-1:1);
end  % nodd
 hmax = max( abs(hn) ) ;
 scale_ww = 1. / (hmax^2);
% normalized hn is the normalized Wavelet response
hn = hn/ hmax;
%===========================================================
% STEP 4.3 PASSBAND RIPPLE "xripple" AND
%          STOPBAND ATTENUATION "xstop"
%====
%Fourier transform of hn & hn in the next channel
ich = 0;
arg_rot = twopi* rem( (0:N-1)*ich /nc , 1 );
[freq, hw_db] = freq_rsp(hn, arg_rot, n_fft);
%        hn_data = [hn_data hn'];
%        hw_data = [hw_data hw_db'];
%====
%===== peak_to_peak ripple in passband
max_ripple = max( hw_db(1: nfft_pass+1));
min_ripple = min( hw_db(1: nfft_pass+1));
xripple = max_ripple - min_ripple ;
%=== stopband atttenuation
  xstop = max(hw_db(nfft_stop+1:nfft_stop+nfft_pass+1) );
%===========================================================
```

Replacement Sheet

FIG. 5E

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```
% STEP 5 WEIGHTED LS ERROR METRICS FOR:
%        - J(PASSBAND) = "beta_pass"
%        - J(STOPBAND) = "beta_stop"
%        - J(DEADBAND) = "beta_dead"
%==========================================================
  err_pass = b_vector' * passband * b_vector;
  err_stop = b_vector' * stopband * b_vector;
  err_dead = b_vector' * deadband * b_vector;
  beta_pass = alpha_1 * err_pass;
  beta_stop = alpha_2 * err_stop;
  beta_dead = alpha_5 * err_dead;
%==========================================================
```

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```
% STEP 6 ISI AND ACI LS:
%    - MATRICES "w_matrix" AND "w_f_matrix"
%    - METRICS  J(ISI)="errM_isi" AND J(ACI)="errM_aci"
%    - SNR ERROR CONTRIBUTORS "errV_isi" AND "errV_aci"
%==========================================================
% STEP 6.1 J(ISI):
%             - LS ERROR MATRIX "w_matrix"
%             - J(ISI) = "errM_isi"
%             - SNR LOSS ISI ERROR "errV_isi"
%=====
for k_wave = 0:M
  n_i = N - 1 - k_wave*nc;
  w_vector(k_wave+1) = 0.;
  for ii = 0:n_i
% ISI error residual vector w_vector
  w_vector(k_wave+1)=w_vector(k_wave+1)+hn(ii+1)*hn(ii+1+ ...
                   nc*k_wave);
 end
end
scale_isi_aci = 1/w_vector(1) ;
w_vector = w_vector * scale_isi_aci;      % normalize
errV_isi = sum(w_vector(2:M) .*w_vector(2:M)); %ISI LS error
%2-sided power summation of isi residual errors
      errV_isi = 2. * errV_isi;
      errV_isiMax = max( abs(w_vector(2:M)) );
 %=====a_matrix = m+1  x 2m+1 = A
a_matrix= zeros(m+1,2*m+1);
for i_r = 0:m
  n_cc = i_r * nc;
  if ( i_r>=1 & i_r<=M)
      nic = (n_cc+1):(2*m+1);
      a_matrix(i_r+1, nic) = hn(nic  - n_cc) ;
  end
end
%====
```

Replacement Sheet

FIG. 5E

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```
% STEP 5 WEIGHTED LS ERROR METRICS FOR:
%        - J(PASSBAND) = "beta_pass"
%        - J(STOPBAND) = "beta_stop"
%        - J(DEADBAND) = "beta_dead"
%============================================================
  err_pass = b_vector' * passband * b_vector;
  err_stop = b_vector' * stopband * b_vector;
  err_dead = b_vector' * deadband * b_vector;
  beta_pass = alpha_1 * err_pass;
  beta_stop = alpha_2 * err_stop;
  beta_dead = alpha_5 * err_dead;
%============================================================
```

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```
% STEP 6 ISI AND ACI LS:
%    - MATRICES "w_matrix" AND "w_f_matrix"
%    - METRICS  J(ISI)="errM_isi" AND J(ACI)="errM_aci"
%    - SNR ERROR CONTRIBUTORS "errV_isi" AND "errV_aci"
%============================================================
% STEP 6.1 J(ISI):
%             - LS ERROR MATRIX "w_matrix"
%             - J(ISI) = "errM_isi"
%             - SNR LOSS ISI ERROR "errV_isi"
%=====
for k_wave = 0:M
  n_i = N - 1 - k_wave*nc;
  w_vector(k_wave+1) = 0.;
  for ii = 0:n_i
% ISI error residual vector w_vector
  w_vector(k_wave+1)=w_vector(k_wave+1)+hn(ii+1)*hn(ii+1+ ...
                    nc*k_wave);
 end
end
scale_isi_aci = 1/w_vector(1) ;
w_vector = w_vector * scale_isi_aci;      % normalize
errV_isi = sum(w_vector(2:M) .*w_vector(2:M)); %ISI LS error
%2-sided power summation of isi residual errors
      errV_isi = 2. * errV_isi;
      errV_isiMax = max( abs(w_vector(2:M)) );
 %=====a_matrix = m+1  x 2m+1 = A
a_matrix= zeros(m+1,2*m+1);
for i_r = 0:m
  n_cc = i_r * nc;
  if ( i_r>=1 & i_r<=M)
      nic = (n_cc+1):(2*m+1);
      a_matrix(i_r+1, nic) = hn(nic  - n_cc) ;
  end
end
%====
```

FIG. 5G

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```
% STEP 7  - WEIGHTED LS ERROR METRICS
%         - UPDATE LS ERROR MATRIX "pw_t" FOR NEXT
%           ITERATION
%==========================================================
% STEP 7.1 WEIGHTED LS ERROR METRICS FOR ISI, ACI, TOTAL
%                - WEIGHTED ISI LS ERROR METRIC   "beta_isi"
%                - WEIGHTED ACI LS ERROR METRIC   "beta_aci"
%                - TOTAL LS ERROR METRIC J =  "errM_LS"
%====
beta_isi = alpha_3*errM_isi;
beta_aci = alpha_4*errM_aci;
errM_LS = beta_pass+beta_stop+beta_dead+beta_isi+beta_aci;
%==========================================================
% STEP 7.2 SAVE WEIGHTED LS ERROR METRICS FOR EACH ITERATION
%====
if i_iteration==1
   scale_err = errM_LS;
end
beta_pass_1 = beta_pass * 1./errM_LS;  % in fraction
beta_stop_1 = beta_stop * 1./errM_LS;  % in fraction
beta_dead_1 = beta_dead * 1./errM_LS;  % in fraction
beta_isi_1  = beta_isi  * 1./errM_LS;  % in fraction
beta_aci_1 =  beta_aci  * 1./errM_LS;  % in fraction
%errM_LS = errM_LS / (alpha_1+alpha_2+alpha_3+alpha_4+alpha_5);
errM_LS = errM_LS /scale_err;
err_LS  =[err_LS ; i_iteration ....
beta_pass_1 beta_stop_1 beta_dead_1 beta_isi_1 beta_aci_1  errM_LS];
%==========================================================
% STEP 7.3 UPDATE J LS ERROR MATRIX "pw_t" FOR NEXT ITERATION
%====
p_t  = p_total+ alpha_3 * w_matrix+ alpha_4 * w_f_matrix ;
pw_t =  bw_matrix'*p_t*  bw_matrix;
%==========================================================
```

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```
% STEP 8 SIGNAL-TO-NOISE SNR LOSS
%          - PASSBAND RIPPLE LOSS "xloss_ripple", dB
%          - ISI LOSS             "xloss_isi",    dB
%          - ACI LOSS             "xloss_aci".    dB
%          - TOTAL LOSS           "xloss_total",  dB
%====
% STEP 8.1 SNR LOSSES DUE TO PASSBAND RIPPLE, ISI, ACI, AND
%          THE TOTAL SNR LOSS
%=====     passband ripple loss
x_delta  = 10.^( xripple/2. /20.) - 1.;
xloss_ripple = -10. * log10( 1.0 - x_delta^2 );
%=====     isi loss
xebno  = 10.^( ebno / 10.0 );
xx_isi = xebno  * errV_isi ;
xloss_isi = 10. * log10( 1.0 + xx_isi );
%====
```

Replacement Sheet

FIG. 5H

```
%====    aci loss
x_g_aci   = 10.^( x_imbal_aci / 10. ) ;
xx_aci    = xebno * errV_aci * x_g_aci ;
%====
xloss_aci = 10. * log10( 1. + xx_aci );
% ====   total loss
xloss_total =  10.* log10( 1.+ xx_isi + xx_aci ) + xloss_ripple;
%============================================================
% STEP 8.2 SAVE SNR LOSSES FOR EACH ITERATION
%====
loss_LS  =[loss_LS ; i_iteration  xloss_total ....
      xloss_ripple  xloss_isi  xloss_aci ];
%====
%==== end of iterations
end
%============================================================
```

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```
% STEP 9 WAVELET DESIGN FOR FIG. 6
%============================================================
% STEP 9.1 WAVELET FREQUENCY DESIGN HARMONICS "hn_eig
%====
'Harmonic number  Harmonic value'
[(0:n_f-1)'         hw_eig]
%============================================================
        0    0.9651
   1.0000    0.9499
   2.0000    0.9842
   3.0000    0.9485
   4.0000    0.9869
   5.0000    0.9434
   6.0000    1.0000
   7.0000    0.8428
   8.0000    0.2266
   9.0000   -0.0018
  10.0000    0.0019
  11.0000   -0.0006
  12.0000    0.0003
  13.0000    0.0001
  14.0000   -0.0000
  15.0000    0.0002
%============================================================
% STEP 9.2 WAVELET TIME RESPONSE "hn"
%====
'Sample index   Wavelet response'
[(0:m)'         hn(m+1:2*m+1)']
%============================================================
        0    1.0000
   1.0000    0.9941
   2.0000    0.9765
   3.0000    0.9476
   4.0000    0.9080
   5.0000    0.8586
```

Replacement Sheet

FIG. 5I

6.0000	0.8004
7.0000	0.7347
8.0000	0.6627
9.0000	0.5860
10.0000	0.5062
11.0000	0.4248
12.0000	0.3434
13.0000	0.2635
14.0000	0.1867
15.0000	0.1141
16.0000	0.0471
17.0000	-0.0135
18.0000	-0.0666
19.0000	-0.1117
20.0000	-0.1485
21.0000	-0.1766
22.0000	-0.1961
23.0000	-0.2073
24.0000	-0.2106
25.0000	-0.2065
26.0000	-0.1959
27.0000	-0.1795
28.0000	-0.1583
29.0000	-0.1335
30.0000	-0.1060
31.0000	-0.0769
32.0000	-0.0474
33.0000	-0.0182
34.0000	0.0095
35.0000	0.0350
36.0000	0.0577
37.0000	0.0769
38.0000	0.0923
39.0000	0.1035
40.0000	0.1106
41.0000	0.1134
42.0000	0.1121
43.0000	0.1071
44.0000	0.0987
45.0000	0.0873
46.0000	0.0736
47.0000	0.0581
48.0000	0.0414
49.0000	0.0242
50.0000	0.0070
51.0000	-0.0096
52.0000	-0.0250
53.0000	-0.0389
54.0000	-0.0507
55.0000	-0.0603
56.0000	-0.0674
57.0000	-0.0719
58.0000	-0.0738
59.0000	-0.0731

Replacement Sheet

FIG. 5J

60.0000	-0.0701
61.0000	-0.0649
62.0000	-0.0578
63.0000	-0.0492
64.0000	-0.0393
65.0000	-0.0287
66.0000	-0.0177
67.0000	-0.0066
68.0000	0.0041
69.0000	0.0141
70.0000	0.0231
71.0000	0.0309
72.0000	0.0372
73.0000	0.0420
74.0000	0.0451
75.0000	0.0465
76.0000	0.0463
77.0000	0.0445
78.0000	0.0414
79.0000	0.0370
80.0000	0.0315
81.0000	0.0253
82.0000	0.0185
83.0000	0.0113
84.0000	0.0042
85.0000	-0.0029
86.0000	-0.0095
87.0000	-0.0155
88.0000	-0.0208
89.0000	-0.0252
90.0000	-0.0287
91.0000	-0.0311
92.0000	-0.0325
93.0000	-0.0328
94.0000	-0.0321

Replacement Sheet

FIG. 5K

```
  113.0000    0.0187
  114.0000    0.0179
  115.0000    0.0168
  116.0000    0.0154
  117.0000    0.0138
  118.0000    0.0121
  119.0000    0.0103
  120.0000    0.0085
  121.0000    0.0067
  122.0000    0.0051
  123.0000    0.0036
  124.0000    0.0024
  125.0000    0.0013
  126.0000    0.0006
  127.0000    0.0001
  128.0000   -0.0000
%============================================================
```

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```
% STEP 10 ITERATION CONVERGENCE IS MEASURED BY THE
%         CONVERGENCE OF THE LS ERRORS IN
%      figure (1), figure(2)
%============================================================
% plots
figure(1)
plot(err_LS(:,1),err_LS(:,7),'k')
legend('Total LS error relative to iteration=1')
ylabel('Total LS error relative to iteration=1')
xlabel('Iteration number')
title('TOTAL LS ERROR J  VS. ITERATION')
grid on
hold on
%====
figure(2)
plot(err_LS(:,1),err_LS(:,2),'k')
hold on
plot(err_LS(:,1),err_LS(:,3),'k--')
plot(err_LS(:,1),err_LS(:,4),'k')
plot(err_LS(:,1),err_LS(:,5),'b')
plot(err_LS(:,1),err_LS(:,6),'b--')
title('LS ERROR CONTRIBUTORS VS. ITERATION')
legend('passband','stopband','deadband','ISI','ACI')
ylabel('LS error relative to total=1')
xlabel('Iteration number')
grid on
hold on
%============================================================
```

Replacement Sheet

FIG. 5L

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```
% STEP 11 PARAMETERS ARE SELECTED TO OPTIMIZE:
%      - WAVELET FILTER PERFORMANCE IN figure(3)
%      - WAVELET RIPPLE, ISI, ACI  SNR LOSSES IN figure(4)
%      - WAVELET TIME RESPONSE IN figure(5)
%==============================================================
figure(3)
plot(freq*M, hw_db,'k')
axis([0 200 -100 10])
grid on
xlabel('Frequency/Wavelet sample rate')
ylabel('Power Spectrum, dB')
hold on
x2=length(hw_ref);
x3=length(hw_wsr);
plot(freq(1:x2)*M,hw_ref, 'b--')
plot(freq(1:x3)*M,hw_wsr, 'b-.')
legend('Wavelet response', 'pass & stop templates', 'Wavelet sample
rate')
title('WAVELET FREQUENCY RESPONSE')
grid on
axis([0 1.4 -100 0])
hold on
%====
figure(4)
plot(loss_LS(:,1),loss_LS(:,2),'k')
hold on
plot(loss_LS(:,1),loss_LS(:,3),'k--')
plot(loss_LS(:,1),loss_LS(:,4),'b')
plot(loss_LS(:,1),loss_LS(:,5),'b--')
title('SNR LOSS VS. ITERATION')
legend('total','ripple','ISI','ACI')
ylabel('SNR LOSS, dB')
xlabel('Iteration number')
grid on
hold on
%====
figure(5)
xx=(-m:m)';
xx=xx/M;
plot(xx,hn','k')
hold on
xlabel('Time/Wavelet sample rate')
ylabel('Wavelet time response')
hold on
title('WAVELET TIME RESPONSE')
grid on
axis([-8 8 -0.4 1])
%==============================================================
```

Replacement Sheet

FIG. 5M

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```
% STEP 12 CALCULATION OF NEW WAVELET WAVEFORM "hn_new"
%           FOR THE PARAMETERS:
%           - "p"  SCALE (DILATION)
%           - "q"  TRANSLATION
%           - "k"  FREQUENCY
%==============================================================
% Wavelet parameters
p = 2  % scale change or dilation
q = 2  % time translation
k = 3  % frequency translation
%==============================================================
% STEP 12.1 WAVELET SAMPLE INTERVAL "M_new" AND LENGTH "N_new"
%====
%== Wavelet sample interval M_new for:

%     Case 1: Fix M_new = M and dilate sampling
%       hn = hn(n 2^-p - q M)
%       n_new = n 2^-p
%             = n_p for n = n_0 + n_p 2^-p
%
%   Case 2: Fix sampling and dilate M_new = 2^p M
%       hn = hn(n - q M_new)
        M_new = M*(2^p);
%====
N_new = M_new*L+1;  % Wavelet length
%====
nodd= fix( N_new/2 );
nodd = N_new - 2 * nodd ;
if ( nodd == 1)
  m_new = (N_new - 1 ) /2   %  N is odd
else
   m_new = N_new/2 ;   %    N is even
end
%==============================================================
% STEP 12.2 MATRIX "bw_matrix_new" FOR MAPPING WAVELET
%           FREQUENCY DESIGN HARMONICS INTO NEWWAVELET IMPULSE
%           RESPONSE IN TIME
%====
bw_matrix_new = zeros(m_new,n_f);
for i_r=1:m_new        % freq
   ang = 2*pi* rem( (i_r)*(0:n_f-1)/(N_new-1),1); % time
   bw_matrix_new(i_r+1, :) = 2 * cos(ang);
end
bw_matrix_new(1,:) = ones(1,n_f);
%==============================================================
% STEP 12.3 MAP WAVELET FREQUENCY DESIGN HARMONICS "hw_eig"
%           INTO NEW WAVELET IMPULSE RESPONSE IN TIME "hn_new"
%====
%==== hn_0 = hn_new without translations in time & frequency
hw_eig2    = hw_eig;
hw_eig2(1) = 0.5*hw_eig(1);
%====
```

Replacement Sheet

FIG. 5N

```
b_vector = bw_matrix_new * hw_eig2;
if ( nodd == 1)    % N_new is odd
   hn_0(1:m_new) = 0.5*b_vector((m_new+1):-1:2);
   hn_0(m_new+1) = b_vector(1);
   hn_0(m_new+2:N_new) = hn_0(m_new:-1:1);
else               % N_new is even
   hn_0(m_new:-1:1)= 0.5 * b_vector(1:m_new);
   hn_0( m_new+1:1:2*m) = hn_0(m_new:-1:1);
end  % nodd
hmax_0 = max( abs(hn_0) ) ;
%==== normalized hn_0 is new baseband Wavelet with q=k=0
hn_0 = hn_0/ hmax_0;
%==== hn_1 is hn_0 with translation in time q*M_new
for n=1:N_new+q*M_new
   if n <= q*M_new
      hn_1(n) = 0;
   else
      hn_1(n) = hn_0(n-q*M_new);
   end
end
%====
%==== hn_new is hn1 with translation in frequency by k
for n=1:N_new+q*M_new
   hn_new(n) = hn_1(n)*exp( i*(2*pi*k*(n-1)/(M_new*L)) );
end
%============================================================
% STEP 12.4 PLOT WAVELET TIME RESPONSE FOR:
%         - MOTHER WAVELET "hn"
%         - NEW WAVELET "hn_new" WITHOUT FREQUENCY TRANSLATION
%====
figure(6)
xx1 = (L/2)*(1-1/2^p)*M_new ;
xx2 = (L/2)*(1+1/2^p)*M_new ;
for n=1:N_new+q*M_new
   if n<xx1 | n>xx2
      hn1(n) = 0;
   else
      hn1(n) = hn(n-xx1+1);
   end
end
x_n = (1:N_new+q*M_new)/M_new;
x_n = x_n-L/2;
plot(x_n,hn1,'k')
hold on
plot(x_n,hn_1,'k--')
grid on
hold on
legend('MOTHER WAVELET','NEW WAVELET')
xlabel('Time/hn\_new sample rate')
ylabel('Wavelet time response')
hold on
title('TIME RESPONSE FOR MOTHER, NEW WAVELETS')
%============================================================
```

Replacement Sheet

FIG. 50

```
% STEP 12.5 PLOT WAVELET FREQUENCY RESPONSE FOR:
%           - MOTHER WAVELET "hn"
%           - NEW WAVELET "hn_new"
%           vs. frequency/hn     sample rate
%           vs. frequency/hn_new sample rate
%============================================================
figure(7) % vs. frequency/hn sample rate
ich = 0;
arg_rot = twopi* rem( (0:N-1)*ich /nc , 1 );
[freq, hw_db] = freq_rsp(hn, arg_rot, n_fft);
plot(freq*M, hw_db,'k')
hold on
ich=k;
arg_rot = twopi* rem( (0:N_new-1)*ich /M_new , 1 );
[freq, hw2_db] = freq_rsp(hn_0, arg_rot, n_fft);
plot(freq*M,hw2_db,'k--')
axis([0 8 -100 10])
grid on
legend('MOTHER WAVELET','NEW WAVELET')
xlabel('Time/hn sample rate')
ylabel('Wavelet time response')
hold on
title('POWER SPECTRUM OF MOTHER, NEW WAVELETS')
xlabel('Frequency/hn sample rate')
ylabel('Power Spectrum, dB')
%====
%==== plot frequency response of hn, hn_new
%     vs. frequency/hn_new sample rate
figure(8)
plot(freq*M_new, hw_db,'k')
hold on
plot(freq*M_new,hw2_db,'k--')
axis([0 8 -100 10])
grid on
legend('MOTHER WAVELET','NEW WAVELET')
xlabel('Time/hn sample rate')
ylabel('Wavelet time response')
hold on
title('POWER SPECTRUM OF MOTHER, NEW WAVELETS')
xlabel('Frequency/hn\_new sample rate')
ylabel('Power Spectrum, dB')
%============================================================
```

Replacement Sheet

FIG. 5P

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```
% STEP 13 FUNCTIONS USED IN MATLAB PROGRAM
%==========================================================
% STEP 13.1 FUNCTION "pmn" COMPUTES MATRIX FOR J(BAND) IN
%          EQ. 23
%====
function p_matrix= pmn(omega_l,omega_u, N,an)
%==========================================================
%     compute the real, symmetric, and positive definite matrix
%     input:    omega_l: lower edge (radians)
%               omega_u: upper edge (radians)
%               N: filter length,     an(.): 1xm column vector
%     output
%      p_matrix(n,m): a nXm real,symmetry and positive-definite matrix
%==========================================================
 twopi = 2. * pi;
%     check filter lenght is odd or even
 nodd = fix(N/2);
 nodd = N - 2 * nodd;
 if ( nodd == 1)
    m = ( N-1) /2;   %    filter length 'N' is odd
  else
   m = N/2 ;             %    N is even
  end
if ( nodd == 1)
  for n= 0:m
    for m1= 0:m
      if ( m1 ==n )
          if ( n ~= 0)
p_matrix(n+1,m1+1)=1./pi*((an(n+1)*an(n+1)+0.5)*(omega_u-omega_l)-....
2.* an(n+1) * ( sin( n*omega_u )- sin( n*omega_l ) ) ....
/n + (sin(2.* n*omega_u) - sin(2.*n*omega_l))/( 4.* n) );
          else
p_matrix(n+1,m1+1)=1./pi*(an(n+1)-1.)*(an(n+1)-1.)*(omega_u-omega_l);
           end
        else
          if ( m1 ~= 0 & n ~= 0)
p_matrix(n+1,m1+1)=1./pi*(an(n+1)*an(m1+1)*(omega_u-omega_l)....
-( an(m1+1) *m1*( sin( n*omega_u ) - sin( n*omega_l ) )+  ....
an(n+1)  *n *( sin(m1*omega_u) -  sin(m1*omega_l ) ) )/( m1*n ) + ( ...
(n+m1)* (  sin( (n-m1)*omega_u ) -sin((n-m1)*omega_l)) ...
+(n-m1)*(sin( (n+m1)*omega_u )-sin((n+m1)*omega_l)))/(2.*(n*n -m1*m1)));
           else
              if (n == 0)
p_matrix(n+1,m1+1)=1./pi*(an(n+1)-1.)*(an(m1+1)*(omega_u-omega_l)-....
( sin(m1*omega_u )-sin( m1*omega_l) ) /m1 );
             end
             if (m1 == 0)
p_matrix(n+1,m1+1)=1./pi*(an(m1+1)-1.)*(an(n+1)*(omega_u-omega_l)-.....
( sin(n*omega_u )-sin(n*omega_l) ) /n );
            end
          end
          end
  end % end of m1 loop
 end  % end of n loop
else
%====
```

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Replacement Sheet

FIG. 5Q

```
% =====    when N is even  ========
   for n = 0:m-1
     for m1 = 0:m-1
        if ( m1 == n )

p_matrix(n+1,m1+1) = 1./pi * (  ...
( an(n+1)*an(n+1) + 0.5) * ( omega_u - omega_l ) - ...
2. * an(n+1) * ( sin( (n+.5) * omega_u) -  ...
sin( (n+.5) * omega_l) )/( n + 0.5 ) +  ...
( sin( (2*n+1) * omega_u ) -sin( (2*n+1) * omega_l ) ) ...
/( 2. * ( 2.*n + 1) )  )  ;
          else
p_matrix(n+1,m1+1) = 1./pi * (  ...
an(n+1) * an(m1+1) * ( omega_u - omega_l ) - ...
an(m1+1)*(sin((n+.5)*omega_u)-sin((n+.5)*omega_l))/( n + 0.5) - ...
an(n+1)*(sin((m1+.5)*omega_u)-sin((m1+.5)*omega_l))/(m1+0.5)+ ...
(sin( (n-m1)*omega_u) - sin( (n-m1)*omega_l) ) /(2.* (n-m1)) +  ...
(sin(( n+m1+1)*omega_u)-sin(( n+m1+1)*omega_l))/(2.*(n+m1+1)));
          end
      end % end of m1 loop
   end  % end of n loop
end    % end of  if nodd =1
%============================================================
% STEP 13.2 FUNCTION "freq_rsp" COMPUTES FOURIER TRANSFORM OF
%           INPUT "hn" VS.  FREQUENCY/WAVELET SAMPLE RATE
%====
function [freq, hw_db] = freq_rsp(hn, arg_rot, n_freq )
% Fourier transform of input hn
% in normalized freq interval (0., 0.5)
% frequency response
% n_freq # of frequency
twopi = 2. * pi;
df = 0.5/ (n_freq -1) ;
n_filter = length(hn);
m=(n_filter-1)/2;
freq = (0:df:0.5);
for nf = 1: n_freq
  arg=twopi * rem( freq(nf) * ((1:n_filter) -1-m),1);
  hw = sum( hn .*exp( (-arg+arg_rot)*i) );
  hw_mag(nf) = abs( hw);
end
hw_max = max( abs(hw_mag) );
hw_mag = hw_mag /hw_max;
hw_db = 20. * log10( hw_mag+ 1.e-20);
%============================================================
```

FIG. 6 PSD for Wavelet and Square-Root Raised Cosine

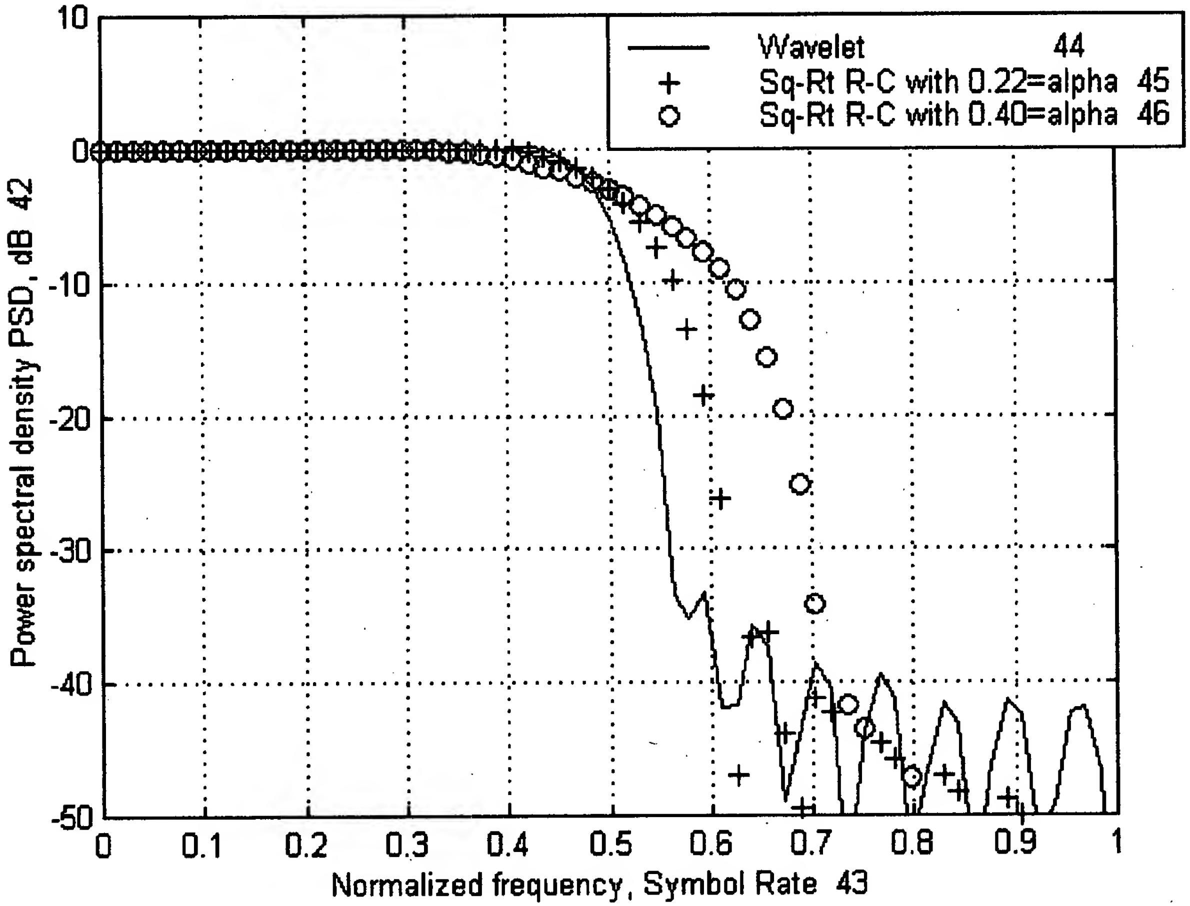
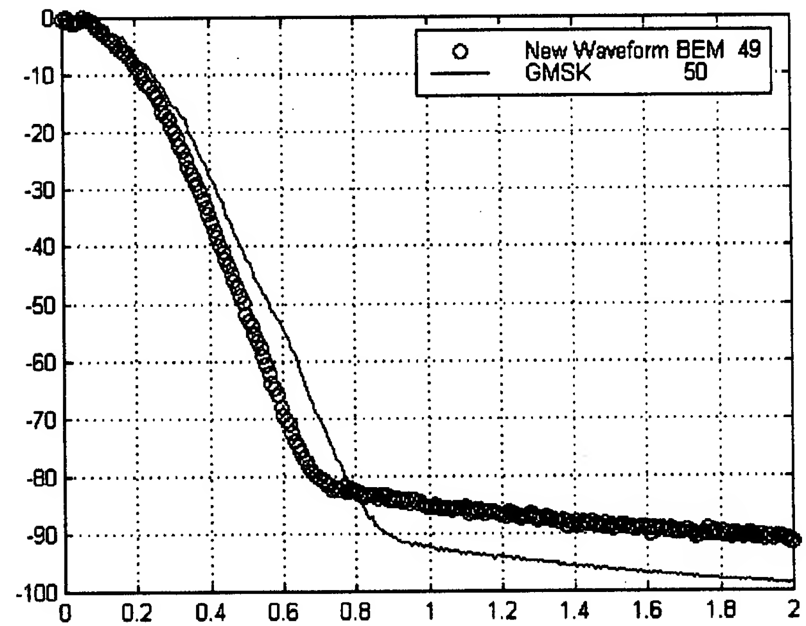


FIG. 7 PSD for New Waveform BEM and GMSK

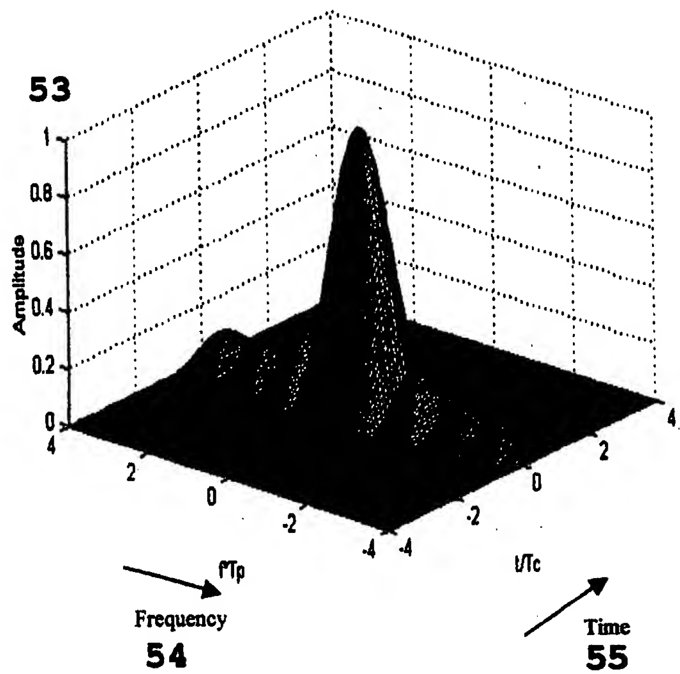
Power spectral density PSD, dB 47



Normalized frequency, bit rate 48

FIG. 8 Radar Ambiguity Functions of Wavelet and Unweighted Chirp Waveform

Wavelet Ambiguity Function 51



Unweighted Chirp Ambiguity Function 52